PROPERTIES OF THE FORMATION AND MOTION OF VORTEX RINGS IN WATER

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Results are given of an experimental study of the formation and motion of vortex rings in water. The dependence of the velocity of the vortices' motion and their dimensions on the distance they travelled was obtained with frame-by-frame and high-speed photography. The rate of exchange of liquid trapped by the vortex at the time of its formation with the liquid medium in which it moves was determined by photometric scanning of the negative image of the vortex. The experimental results are compared with available theoretical representations of the character of the motion of a vortex ring [1-3].

1. Experimental Apparatus and Procedure for Making Measurements

The experiments were performed with apparatus whose schematic form is shown in Fig. 1. The vortex generator 1, mounted in the floor of a plastic vessel 2, was a cylinder of 35 mm inside diameter, divided by a piston 3. When the piston moved between the lower and upper stops 5, 4, a fixed quantity of liquid was ejected into the vessel through the generator nozzle with diameter 19.3 mm forming a vortex ring. In all experiments the distance travelled by the piston was 4 mm. A rubber bulb 8 was connected with the bottom part of the generator housing through a tube 7, so that the velocity at which the liquid jet was ejected through the generator nozzle could be determined by the pressure on the bulb.

Before the experiment, to make the propagation of the vortex visible, a dye solution was slowly introduced into the generator through an aperture 9. The specific weight of the solution exceeded that of the pure water by a small amount. Thus, with sufficiently slow input of the dye, the pure water was forced out through the nozzle of the generator and a sharp interface between the dye and the pure water was maintained. The degree of dilution of the dye was chosen so that during photometric scanning of the negatives the signal level would not fall outside the operating range of the characteristic of the photoelements, and so that photometric scanning of the vortices would reveal their internal structure.

Frame-by-frame photography of the vortices was performed with the use of a specially constructed multiple-objective camera. The housing of the camera was a casing 10 divided into ten frame compartments by opaque partitions 11, the frame size being 55×55 mm^2. The camera had a compartment 12 for the storage of unexposed film (rolls of amateur film 60 mm in width, extended by light-shielding strips, were used) and a compartment 13 where the exposed film was rolled onto a spool 14 with light guards in the form of wide flanges. The film channel was covered with a lid 15 having cleats 16 to tighten the film and a peephole 17 for observation of the number of frames pulled through.

Ten type T-22 objectives 18 (focal length 75 mm) with central shutters were mounted in the camera. Focusing of the images was accomplished by adjusting the telescoping tubes 19.

The objective shutters were set for a 1/250 sec exposure. As the flow velocity did not exceed 25 cm/sec in even the most rapidly moving vortices at the moment they were photographed, the maximum spread of the image on the film was no more than 1 mm, which was sufficiently small for the required resolution of the image. Successive activation of the shutters was synchronized with the passage of the vortex past the optical axes of the objectives.

Use of the photographic apparatus described above made it possible to obtain sharp, one-to-one scale negative images during the course of all the vortex's motion over a distance up to 650 mm in identical posi-

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tions relative to the objective in operation. The photography of a vortex in all phases of its motion on one film ensures identical conditions under which all the frames are processed, this being necessary for making accurate quantitative photometric estimates of the mass of dye carried along by the vortex.

Along with the frame-by-frame photography, motion pictures of the flow process during formation of the vortex near the nozzle of the generator were made with a KSR-IM motion picture camera having a frame speed of 36 per second and a high-speed "Pentazet" motion picture camera having a frame speed of 300 per second. The phenomena were illuminated by uniformly lighting a dull white screen behind the plastic vessel with a system of incandescent lamps.

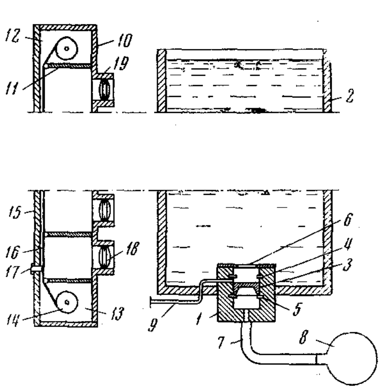

Fig. 1

In order to determine the rate by mass at which liquid was exchanged between the vortex ring and the medium in which it was moving each square millimeter of the negative image of the vortex was measured photometrically, and the resulting signals were then summed over the entire area of the image. It was found that the density of measurement points was sufficient for resolution of characteristics of the flow within the vortex, whose characteristic linear dimensions did not exceed 1 mm.

The problem of identification of the signals obtained by the photoelements with the quantity of dye in the vortex arises from the fact that the density of the optical image on the film depends in a complicated nonlinear way on the concentration of dye and the depth of the region occupied by the dyed liquid. Moreover, this dependence changes with changes in the intensity of illumination, the photographic characteristics of the film emulsion, and the conditions under which the photosensitive layer is processed. Therefore the intensity of illumination, the composition of the dye, and the exposure of, and conditions under which the photographic materials were processed were chosen by preliminary experimentation in such a way that two basic characteristics were close to their linear range: 1) the dependence of the optical density of the negative on the degree of dilution of the dye, the region occupied by it remaining unchanged, and 2) the dependence of the optical density of the negative on the depth of the region occupied by the dye, its composition remaining unchanged. Through use of calibration curves, constructed from photographs of optical polygons, the quantity of dye per unit area of the image was determined in relative units for a known level of background illumination. Out of a great quantity of material available for final processing those films were chosen that had an optical density of image and background illumination that did not fall outside the limits selected for linearity of the characteristics. On average the possible error was 15%. This error increased to 25% for low signal levels, observed only in measurements of mass of dye in rapidly moving vortices near the end of the range of observation.

2. Formation of Vortex Rings and Initial Phase of Their Motion

Successive motion picture frames of the formation and motion of a vortex ring in water are shown in Fig. 2. The set of motion picture frames *a* corresponds to the initial phase of the formation of the vortex ring (time between frames was 150 msec); set b, to the motion of a laminar vortex ring (the initial velocity of the flow in the jet of liquid from the vortex generator was 4 cm/sec); and set c, to the motion of a turbulent vortex ring (initial velocity of the flow in the liquid jet 25 cm/sec). In the last two sets of motion picture frames the vortex travelled about 70 mm between frames.

It is found that in the initial stage of the piston's motion within the vortex generator the velocity of the interface between the pure and the dyed liquid is the same in both the horizontal (along the upper plane of the generator housing) and the vertical directions (first and second frames of Fig. 2a). This velocity increases rapidly and attains its maximum within 20 to 50 msec. But then, even though the rate of flow of liquid from the generator remains constant, the velocity at which the interface moves along the axis of the generator begins to diminish.

The dependence of the relative velocity V/v_0 of the interface along the generator axis on the relative distance X/δ travelled by the interface is shown in Fig. 3a, where v_0 is the maximum velocity of the interface and δ is the diameter of the exit aperture of the vortex generator. Here, and in Figs. 3b and 3c, the experimental points 1, ..., 8 correspond to the following initial values of the exit velocity of the jet at the vortex generator (cm/sec)

	1	2	3	4	5	6	7	8
$v_0 =$	4.3	2.9	11.9	13.0	13.9	19.5	26.0	27.0

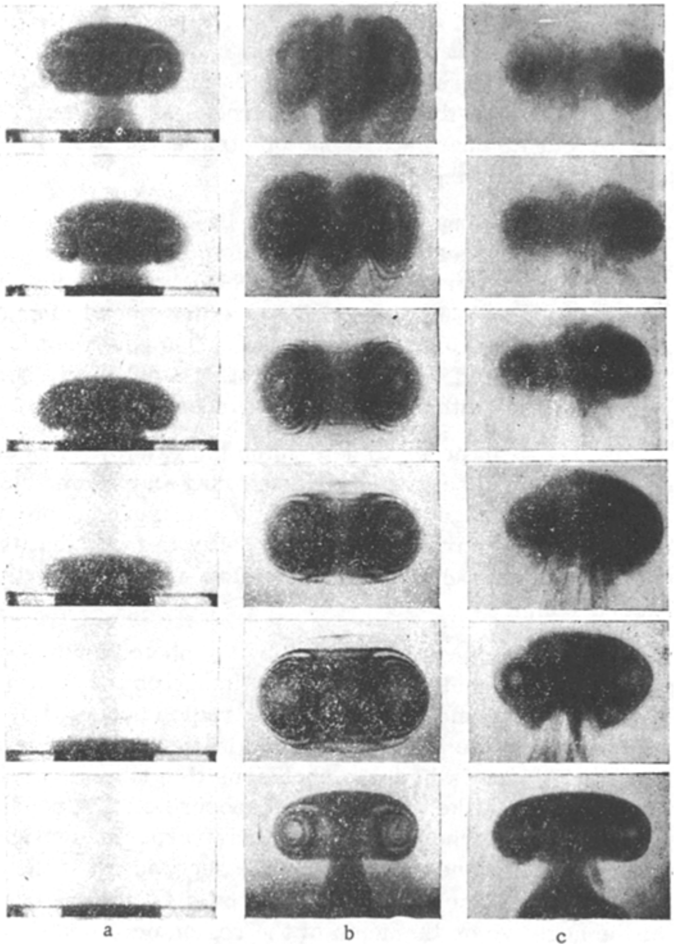

Fig. 2

As can be seen from the plot, the maximum velocity v_0 is attained when the interface between the pure and the dyed liquid has travelled a distance 0.4 δ from the mouth of the generator nozzle. At this time the leading part of the jet takes the form of a "rivet" (third frame), and the decrease in the velocity of the motion of the jet along the generator axis is explained by the fact that a radial flow develops along the entire front surface of the jet.

As a result of this flow a layer is formed of the dyed and pure liquids, spreading out to the lateral rim of the jet. The motion of this layer is horizontal at first, and then the layer bends down, enclosing a quantity of the pure liquid in the annular region adjacent to the upper flange of the generator housing. Thus, the kernel of the future vortex ring is formed in the immediate vicinity of the exit aperture of the generator, the diameter of the kernel exceeding that of the nozzle by 10-15%.

Figure 3b shows results of the measurement of the diameter d/δ of the jet of dyed liquid during its widening phase (1) and the diameter a/δ of the annular zone forming the kernel of the vortex (2) as functions of the relative distance X/δ. The experiments showed that the character of the variation in the parameters d and a does not depend on the initial flow velocity of the liquid jet. In only a few experiments a slight decrease (up to 10%) in d and a was recorded, which may be explained by pulsations in the rate of flow of liquid out of the nozzle of the vortex generator. In such cases, however, d and a returned rapidly to the values they attained at $X = 0.4\,\delta$. Thus the maximum value v_0 of the velocity of the liquid interface measured in the experiments, which was practically equal to the maximum flow velocity in the jet of liquid out of the generator, and the maximum stable values d_0 and a_0, characterizing the dimensions of the jet at the moment the vortex is finally formed, can be taken as the reference parameters determining the initial velocity and the initial dimensions of the vortex.

The velocity v_x of the motion of the vortex kernel increases much more slowly than V (Fig. 3c). The vortex kernel begins to detatch itself from the top cap of the vortex generator only after the development of a sizeable radial flow in the jet of ejected liquid. When the upper points of the liquid interface reach a

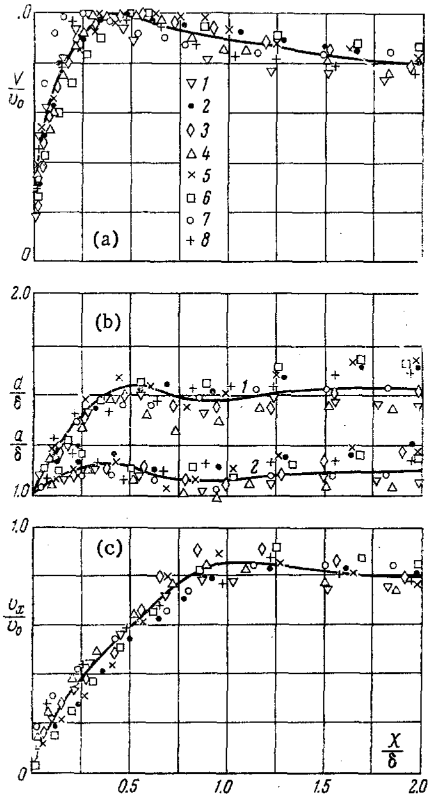

Fig. 3

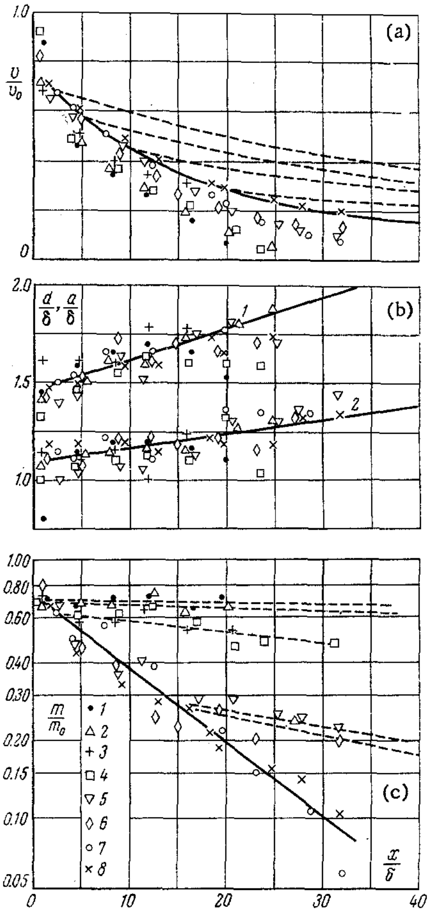

Fig. 4

distance $0.8\ \delta$ (fourth frame of Fig. 2a) the velocity v_x of the kernel's motion attains its maximum value and becomes equal to $0.85\ v_0$. It should be remarked that it is found that at this time the velocity V also equals $0.85\ v_0$ and thereafter v_x and V coincide. At the value of X/δ cited, the flow in the head of the jet is completely formed, and the flow of pure liquid directed toward the axis of the generator, and propagating along its upper plane closes the vortex, separating it from the remainder of the jet of dyes liquid (fifth and sixth frames).

Under the conditions studied, final formation of the vortex occurred when the upper boundary of the jet was at a distance 2δ from the generator nozzle. At this moment the vortex kernal was at a distance $1.6\ \delta$. In the experiments we are describing an intense intercepting flow comes into being when the piston in the cylinder of the generator comes to the end of its course against the upper stop and ceases to eject liquid from the generator. It is possible that the process of vortex formation would continue for a longer time if the emission of the jet were not cut off. In such a case the quantities d_0/δ and a_0/δ, characterizing the initial parameters of the vortex, might exceed the measurements reported in the present paper.

The general pattern of vortex ring formation, described above, is also observed when a vortex is produced in air [2]. However, if the emission velocity of the jet is increased to the extent that shock waves are formed at the mouth of the generator nozzle [4], then the character of the flow of the fluid near the nozzle changes, and it is found that the parameters of the vortex produced are not the same as those studied in this investigation. In our experiments if the time for the velocity of the flow of liquid to grow to its maximum value was shortened to 10 msec, a cavitation pocket formed at the edge of the generator nozzle. Then a ring-shaped cavity with a channel diameter of 2-4 mm was picked up by the vortex and carried along for a considerable distance. However, the dimensions and velocity of the vortex were different from those observed in the case of nonseparating emission of liquid from the nozzle.

A significant result of our investigation was the determination of the limiting value R_0 of the Reynolds number introduced as d_0v_0/ν, at which the vortex ring becomes turbulent. The character of the flow of liquid during stages while the vortex is being formed does not depend on the initial emission velocity of the jet; however, the subsequent flow in the vortex is determined by the magnitude of the initial velocity. Our experiments established that for $R_0 < 2 \cdot 10^3$ a laminar vortex forms and throughout its existence continues to exhibit the spiral structure of layers of pure and dyed liquid (Fig. 2b) that are characteristic of such vortices [2]. For $R_0 > 2 \cdot 10^3$ the spiral structure of the flow that forms initially quickly disappears and the vortex becomes turbulent; however, the external form of the vortex ring is conserved and its nucleus is still visible within (Fig. 2c). It is interesting that for low-speed turbulent vortices the instantaneous value of $R = dv/\nu$ may become less than $2 \cdot 10^3$ on account of the gradual decrease in their velocity. Complete restoration of the laminar flow regime and the apperance of spirally curling layers of liquids was not recorded, but a decrease in isolated eddies and a general quieting of the flow was observed.

3. Measurement of the Velocity and Dimensions of a Vortex Ring during Its Motion over Large Distances

During the course of the motion of a vortex ring a decrease in its velocity occurs, which is greater, the greater its initial velocity. Figure 4a shows the experimentally obtained dependence of v/v_0 on x/δ, where v is the instantaneous velocity of the vortex, v_0 is the maximum emission velocity of the jet of liquid issuing from the generator nozzle, and x is the distance travelled by the kernel of the vortex. Here, and also in Figs. 4b and 4c, the experimental points 1, ..., 8 correspond to the following values of the maximum exit velocity of the liquid jet from the vortex generator (cm/sec):

	1	2	3	4	5	6	7	8
$v_0 =$	5.6	7.0	8.0	8.1	15.4	15.8	23.8	28.4

The experiments showed that after the vortices had travelled a distance of 30δ the loss in their speed amounted to 60–80%. And for small jet emission velocities ($R_0 < 1.5 \cdot 10^3$) the vortex stopped before it reached the 30δ mark, even though the flow continued to circulate in its interior. After a while the vortex spread out and a portion of the dye brought along with it began to sink.

This is explained by the fact that the force of gravity influenced the motion of the slow vortices since the dye solution entrained by them had a specific weight somewhat higher than that of the pure water in the vessel. Thus velocity data for such vortices is not typical. Therefore, despite the fact that points in Fig. 4a corresponding to vortices with different initial velocities do not all lie on one curve, the dependence of v/v_0 on x/δ can be represented by the curve enveloping points obtained for high-speed vortices on which the force of gravity did not have a noticeable influence (solid curve). Special experiments [5], designed to eliminate the influence of gravity in the study of the motion of vortex rings in water have shown that the dependence of v/v_0 on x/δ is the same for vortices with different initial parameters ($10^4 < R_0 < 10^6$) and corresponds to the graph in Fig. 4a.

Simultaneously, with the determination of the velocity of the vortex, measurements were made on two of its linear dimensions: its transverse diameter d and the diameter a of its kernel. In Fig. 4b the dependence (1) of d/δ, and (2) of a/δ, on x/δ is shown.

As seen from the plots, the relative dimensions of the vortices differed from experiment to experiment by 7–10%, but this scatter is not systematic and is not connected with the velocity of the vortex. Only in those cases when the translational motion of the vortex stopped (slow vortices) was there a sharp decrease in the vortex's size and it disintegrated. The dependences of d/δ and a/δ on x/δ obtained experimentally are almost linear and, to an accuracy within ten percent for values of x/δ between 3 and 30, they can be represented by

$$d = 1.46\,\delta + 0.0167x, \qquad a = 1.11\delta + 0.0057x \tag{3.1}$$

The linear character of the dependences (3.1) makes it possible to compare the experimentally measured velocity of a vortex ring with theoretical calculations [6]. In fact, converting the time dependence of the distance L(t) travelled by a turbulent vortex ring

$$L(t) = \frac{a^\circ}{2\alpha}\left[\left(1 + \frac{8\alpha v^\circ}{a^\circ}\, t\right)^{1/4} - 1\right]$$

obtained in a self-similar formulation, and changing to the notation adopted in the present paper, we obtain

$$\frac{v}{v_0}=\frac{v^\circ}{v_0}\left[\frac{2\alpha\delta}{a^\circ}\left(\frac{x}{\delta}-\frac{x^\circ}{\delta}\right)+1\right]^{-3} \tag{3.2}$$

Here x° is some arbitrarily chosen initial coordinate at which the turbulent vortex is considered to be completely formed, v° is the translational velocity of the vortex at the point x°, a° is the diameter of the vortex kernel at the point x°, and a° is the diameter of the vortex kernel at the point x°, and α is the angle of divergence of the vortex kernel, which, according to [6], can be determined as $(a-a^\circ)/2(x-x^\circ)$.

Using the experimental data shown in Fig. 4b, we find that $\alpha = 3.4\cdot10^{-3}$. Then, choosing as the arbitrary initial point the points with the coordinates x° giving $x^\circ/\delta = 2$, 5, 10, and 20 and determining the relative velocities v°/v_0 corresponding to these values of x°/δ from the graph in Fig. 4a and the relative diameters a°/δ of the vortex kernel from the graph in Fig. 4b, we can calculate the subsequent velocity of the vortex on the basis of Eq. (3.2).

Results of the calculation are shown as the dashed curves in Fig. 4a. As the graphs indicate, there is a divergence between the experimental data and the calculations, this divergence becoming more serious, the earlier the stage of the vortex's motion that is chosen as the initial point x°/δ for the calculations. Only for values of x°/δ greater than 20 is it found that the actual trajectory of the vortex's motion is close to the calculated one.

This result is explained by the fact that in the initial stage of the vortex's motion the self-similar character of the vorticity, which is the assumption underlying the formulation of the problem in [6], has not yet had a chance to develop. Moreover, in the experiments described above the initial velocity of even the fastest vortices did not exceed 30 cm/sec ($R_0 < 7\cdot10^3$) and during the course of their motion the instantaneous value of R decreased to $1.5\cdot10^3$. Therefore in the later stages of the vortex's motion the turbulent flow regime that had been formed may disintegrate. It is possible that for sufficiently large initial velocities of the vortices, when $R_0 > 10^4$, and the vortices travel a distance greater than 100δ, the result of a calculation starting with $x^\circ/\delta > 30$ would correspond more closely to experiment. This is confirmed by experiments with high-speed vortices in air [6].

4. Transfer of Impurities by a Vortex Ring

Measurements of the dependence of the relative quantity of dye m/m_0 contained in the vortex on the relative distance x/δ the vortex has travelled are shown in Fig. 4c. Here m is the instantaneous mass of dye contained in the vortex and m_0 is the mass of dye ejected from the generator. The dependences were obtained for vortex rings having various initial velocities giving values of the number R_0 in the range from $1\cdot10^3$ to $7\cdot10^3$.

During formation of the vortex immediately outside the generator 30% of the ejected liquid remains behind. The motion pictures show that this mass is the residue of the jet after the vortex is cut off from it. The fraction of the mass of dye lost at the formation of the vortex does not depend on the emission velocity of the liquid jet from the generator.

However, starting with the distance at which the vortex ring is completely formed and separates from the jet ($x/\delta \sim 2$) the character of further change in the mass of dye contained in the vortex depends appreciably on the initial velocity. If $R_0 < 2\cdot10^3$, i.e., if the vortex is laminar throughout the course of its motion (Fig. 2b), the mass of dye lost is small, and, by the time the vortex reaches a distance of 25δ from the generator it amounts in all to 5-7%.

For large initial velocities of the vortices, when $R_0 > 5\cdot10^3$, a considerable mass of dye is lost all along the measured part of the route travelled by the vortex and amounts to 90-95% at distances of 25δ (Fig. 2c). For turbulent vortices, having initial values R_0 in the range from $3\cdot10^3$ to $5\cdot10^3$, the rate of mass exchange in the initial part of the route is like that for high-speed vortices, but thereafter it drops sharply with the decrease in the instantaneous value of R to $2\cdot10^3$. It is remarkable that it is exactly this value of R that corresponds to the transition from the turbulent to the laminar regime in vortex motion.

Thus, it is found that the relative mass of dye contained in vortices that have travelled the same distance is not the same for vortices with different initial velocities.

For laminar vortices it is natural to suppose that the loss of mass occurs because of convective diffusion. In fact, the viscosity of water ν is 10^{-2} cm²/sec and the coefficient of molecular diffusion D_m in an aqueous solution is about 10^{-5} cm²/sec, whence the Prandtl number $P = \nu/D_m$, characterizing the degree of molecular transfer of matter, has a value of about 10^3. This indicates that for small flow velocities of a liquid the mechanism responsible for the transfer of matter is convective diffusion.

We estimate the rate at which mass of the dye is lost during motion of a vortex in the turbulent regime. The equation for turbulent diffusion has the following form

$$\partial C / \partial t = D_T \Delta C \tag{4.1}$$

where C is the concentration of dissolved substance and D_T is the coefficient of turbulent diffusion.

Assuming that the mixing of the liquid in a turbulent vortex proceeds with sufficient vigor and that some constant distribution of the dye has been established in it, we can make an order-of-magnitude estimate of the magnitude of the terms in Eq. (4.1), replacing the concentration C by the overall instantaneous mass of dye contained in the vortex

$$\frac{\Delta m}{\Delta t} \sim D_T \frac{m}{d^2} \tag{4.2}$$

Dividing the left andright sides of (4.2) by the displacement Δx of the vortex in time Δt, we obtain

$$\frac{\Delta m}{\Delta x} \sim D_T \frac{m}{d^2 v} \tag{4.3}$$

The coefficient of turbulent diffusion D_T can be expressed in terms of quantities characterizing a turbulent flow [7]

$$D_T \sim vd$$

From this it follows that, so long as the vortex remains turbulent, the loss of mass of the dye does not depend on the velocity of the vortex

$$\frac{\Delta m}{\Delta x} \sim -\frac{m}{d} \tag{4.4}$$

In fact, measurements indicate (Fig. 4c, solid curve) that so long as R does not fall below the value $2 \cdot 10^3$, the dependence (4.4) is valid. In accordance with this, after determination of the empirical coefficients, the loss of mass in a turbulent vortex can be described by the formula

$$m = 0.73\, m_0 \exp(-0.064\, x / \delta) \tag{4.5}$$

The coefficient 0.73 in (4.5) is the fraction of the original mass of dye that remain in the vortex after it is completely formed.

It was observed that throughout the motion of laminar vortices (Fig. 2b) the kernel remained "void" and the entire mass of dye was concentrated outside the kernel. In the initial phase of the motion of turbulent vortices (Fig. 2c) the kernel contained primarily liquid from the surrounding medium, but subsequently dye filled the kernel and its loss from the kernel proceeded more slowly than that from all the rest of the volume. Separate experiments showed that after a vortex travels a distance of 150δ it becomes difficult to distinguish it from the surrounding medium [5], as there is practically complete replacement of the liquid contained in the vortex with that of the surrounding medium. Only in the annular region occupied by the vortex kernel the dye is retained somewhat longer.

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